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# Optimization Multiple Responses of Taguchi Method Using PCA-TOPSIS and VIKOR Method

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**Abstract**— Optimizing multiple responses problem has become increasingly relevant issue when more than one product quality characteristics must be assessed simultaneously in a complicated manufacturing process. Two of three Taguchi principles are a specific loss fuction and innovations in experimental design. The loss fuction is to analyse response(s) from the experiment meanwhile the experiment should be robust to other factors outside experimental factors. This study proposed optimization of multiple responses of Taguchi method using PCA-TOPSIS and VIKOR method. The aim of this study was to compare those methods in conditions whether there were correlation between responses. Based on that condition, it would be seen whether the level of correlation among responses affecting the result of PCA-TOPSIS and VIKOR method. The results showed that PCA-TOPSIS and VIKOR method are consistent in producing optimal factor/level combination although there were correlations among responses.

**Index Terms**— Optimal factor/level combination, PCA-TOPSIS method, Taguchi method, VIKOR method, Optimization multi-responses

## 1 Introduction

Manufacturing companies around the world affected by globalisation and are forced to become more competitive with each passing day to maintain their profitability. Total implementation of quality is a principle that is applied as one of the strongest indicators of the competitiveness of the company. Good quality is when the combination of the process parameters in accordance. The optimal combination is obtained by optimization. Optimi-zation is defined as searching activity of variable values that are considered to be optimal, effective, and efficient way to achieve the desired results.

One of Taguchi principle is designing a design which is used frequently due to its effectiveness and efficiency [1]. Taguchi method makes the products or pro-cesses to be robust against the noise, so this method is also known as robust design [3].

Optimization of multiple responses problem has become an increasingly relevant issue when more than one quality characteristic of the product maybe correlated, so they should be assessed simultaneously in a complex manufacturing process. Tong et al [4], used Taguchi multiple respon optimization combined with Principal Component Analysis (PCA) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). TOPSIS is based on the concept in which the best selected alternative has not only the shortest distance from the ideal solution, but it also has the longest distance from negative ideal solution [2]. Ratio of *S/N* in the Taguchi initially used to assess the performance of each response. PCA is then performed on the values of *S/N* to get a collection of components that are not correlated. Finally, relative approximation to the ideal solution resulting from TOPSIS determined as Overall Performance Index (OPI) for multiple responses. PCA-TOPSIS method is an effective and systematic procedure to optimizing multiple responses with Taguchi method due to the ability to solve the correlation problem between responses and reducing computational complexity. PCA-TOPSIS aims to maximize profits.

In addition, the optimization multiple responses of Taguchi method can also be performed using VIKOR method. Tong et al [5] used VIKOR as optimization multiple responses of Taguchi. VIKOR is a method of compromised ranking used for multicriteria decision making (MCDM). MCDM procedure can be used to determine the optimal solution among multiple contrary alternatives and compromising multicriteria. VIKOR (*VlseKriterijumska Kompromisno I Resenje Optimizacija in Serbian*) is applied to decreasing the various measurements of the quality of opposition and compromising response. The optimal combination of factors/level can raise the consistent quality loss is between responses, surely it is not acceptable to customers. Most procedures ignore the diversity in the loss of quality for multiple responses. However, VIKOR method is a systematic procedure to solve this problem. VIKOR method aims to minimize the quality loss.

The aim of this study is to compare the results between the multiple responses of Taguchi method using PCA-TOPSIS and VIKOR method in generating optimal combination of factors/level which conditioned to have different levels of correlation and to see whether the different levels of correlation affect the responses.


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## 2 Research Method

### 2.1 Data

Data used in this study were partly derived from the research of Tong et al [4] and simulation results. Data derived from research consist of 3 responses and 5 factors, with each factor consists of 3 levels. First response characteristic is larger the better ($Y_1$), the second respon is smaller the better ($Y_2$), and the third is larger the better ($Y_3$). In this paper, quadratic model were assumed which the equation of model are:

$$\underline{Y_i} = \boldsymbol{X}\underline{\beta} + \underline{e_i}, i = 1,2,3$$

$$\boldsymbol{X} = [1 \quad \underline{X_1} \quad \underline{X_2} \quad \underline{X_3} \quad \underline{X_4} \quad \underline{X_5} \quad \underline{X_1^2} \quad \underline{X_2^2} \quad \underline{X_3^2} \quad \underline{X_4^2} \quad \underline{X_5^2}]$$

Where $\boldsymbol{X}$ was a model matrix. The combination levels of five factors was called the orthogonal matrix ($L_{18}$) which was adopted from Tong et al [4]. Number 18 refers to the number of experimental runs.

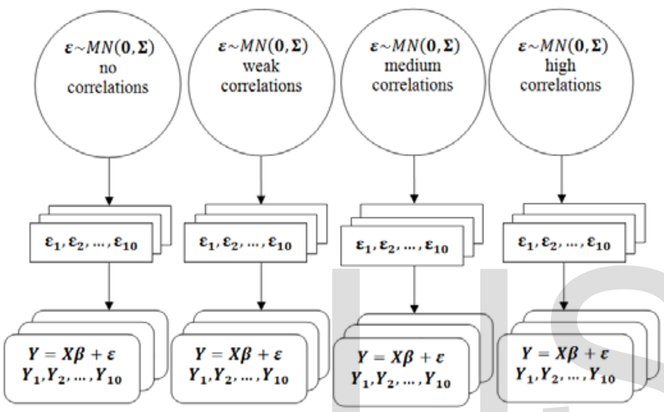


Fig 1. The process of generating data for simulation

Figure 1 shows generating data of the simulation process. Four scenarios were defined: no correlations, weak correlations, medium correlations, and high correlations among responses. Errors were generated based on correlations among responses. Parameter values for each level of the different correlations can be seen in Table 1-3. Table 1 shows parameter regressions when there were no correlations and medium correlations among responses, Table 2 shows parameter regressions when there were weak correlations among responses, and last, Table 3 shows parameter regressions when high correlations were exist among responses.

TABLE 1
Parameter values when
no correlations and medium correlations among responses

| | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| $\beta_0$ | 0.001 | 0.001 | 0.001 |
| $\beta_1$ | 0.002 | 0.002 | 0.002 |
| $\beta_2$ | 0.003 | 0.003 | 0.003 |
| $\beta_3$ | 0.001 | 0.001 | 0.001 |
| $\beta_4$ | 0.002 | 0.002 | 0.002 |
| $B_5$ | 0.003 | 0.003 | 0.003 |
| $\beta_6$ | 0.001 | 0.001 | 0.001 |
| $\beta_7$ | 0.002 | 0.002 | 0.002 |
| $\beta_8$ | 0.003 | 0.003 | 0.003 |
| $\beta_9$ | 0.001 | 0.001 | 0.001 |

TABLE 2
Parameter values when weak correlations among responses

| | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| $\beta_0$ | 0.020 | 0.020 | 0.020 |
| $\beta_1$ | 0.030 | 0.030 | 0.030 |
| $\beta_2$ | 0.040 | 0.040 | 0.040 |
| $\beta_3$ | 0.020 | 0.020 | 0.020 |
| $\beta_4$ | 0.030 | 0.030 | 0.030 |
| $\beta_5$ | 0.040 | 0.040 | 0.040 |
| $\beta_6$ | 0.020 | 0.020 | 0.020 |
| $\beta_7$ | 0.030 | 0.030 | 0.030 |
| $\beta_8$ | 0.040 | 0.040 | 0.040 |
| $\beta_9$ | 0.020 | 0.020 | 0.020 |

TABLE 3
Parameter value when high correlations among responses

| | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| $\beta_0$ | 0.100 | 0.100 | 0.100 |
| $\beta_1$ | 0.200 | 0.200 | 0.200 |
| $\beta_2$ | 0.300 | 0.300 | 0.300 |
| $\beta_3$ | 0.100 | 0.100 | 0.100 |
| $\beta_4$ | 0.200 | 0.200 | 0.200 |
| $\beta_5$ | 0.300 | 0.300 | 0.300 |
| $\beta_6$ | 0.100 | 0.100 | 0.100 |
| $\beta_7$ | 0.200 | 0.200 | 0.200 |
| $\beta_8$ | 0.300 | 0.300 | 0.300 |
| $\beta_9$ | 0.100 | 0.100 | 0.100 |

### 2.2 Methods of Data Analysis

Methodology in this research were:

1) Performing the simulation process as described in Data section.
2) Analyzing the data using the method of PCA-TOPSIS of four scenarios of correlations among responses.
   a. Calculating the ratio *S/N* for each response.
   b. Performing PCA by normalized *S/N* ratio.
   c. Determining the number of selected principal component.
   d. Performing TOPSIS to acquire OPI for multiple responses. Calculating OPI by the formula:

$$C_g = \frac{S_g^-}{S_g^+ + S_g^-}$$

$S_g^-$ and $S_g^+$ can be obtained from:

$$S_g^+ = \sqrt{\sum_{h=1}^{t} (v_{gh} - v_h^+)^2}$$

$$S_g^- = \sqrt{\sum_{h=1}^{t} (v_{gh} - v_h^-)^2}$$

$v_{gh}$ was the weighted quality performance matrix, where the weights were eigen values associated with each principal components. Calculating $v_{gh}$ by the formula:

$$V = \begin{bmatrix} w_1r_{11} & w_1r_{12} & \cdots & w_1r_{1t} \\ w_2r_{21} & w_2r_{22} & \cdots & w_2r_{2t} \\ \vdots & \vdots & & \vdots \\ w_tr_{q1} & w_tr_{q2} & \cdots & w_tr_{qt} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1t} \\ v_{21} & v_{22} & \cdots & v_{2t} \\ \vdots & & \vdots & \vdots \\ v_{q1} & v_{q2} & \cdots & v_{qt} \end{bmatrix}$$

$v_h^+$ were the ideal solution and $v_h^-$ were the ideal negative solution. The ideal solution was maximum of $v_{gh}$ and the ideal negative solution was minimum of $v_{gh}$ ($g$ = 1, 2, ..., $q$ and $h$ = 1, 2, ..., $t$).

e. The process was repeating 10 times. Select the best one among 10 iterations with the most maximum value.

3) Analyzing data using the method of VIKOR of four scenarios of correlations among responses.
   a. Calculating *Quality Loss*
   b. Calculating the normalization *quality loss* (NQL) of each response.
   c. Determining the ideal solution and the ideal negative solution. The ideal solution was minimum of NQL and the ideal negative solution was maximum of NQL.
   d. Calculating the utility measure and regret measure for each response using the following equation:

$$M_i = \sum_{l=1}^{e} \frac{w_l(f_l^+ - f_{il})}{f_l^+ - f_l^-}$$

$$N_i = \operatorname*{Max}_{l} \left[\frac{w_l(f_l^+ - f_{il})}{f_l^+ - f_l^-}\right]$$

   e. Calculating the VIKOR index by the formula:

$$Q_i = v\left[\frac{M_i - M^-}{M^+ - M^-}\right] + (1 - v)\left[\frac{N_i - N^-}{N^+ - N^-}\right]$$

   f. The process was repeating 10 times. Select the best one among 10 iterations with the most maximum value.

4) Comparing the results of optimization multiple responses of Taguchi between PCA-TOPSIS and VIKOR method.

## 3 Result and Discussion

### 3.1 Results of Data Generation Simulation

Table 4 shows the first eigen value of errors ($e$) and responses ($Y$) of four correlations among responses.

TABLE 4
First eigen value of errors ($e$) and responses ($Y$) of four correlations among responses

| No correlations | | Weak correlations | | Medium correlations | | High correlations | |
|---|---|---|---|---|---|---|---|
| $e$ | $Y$ | $e$ | $Y$ | $e$ | $Y$ | $e$ | $Y$ |
| 1.094 | 1.097 | 1.092 | 1.310 | 2.633 | 2.638 | 2.911 | 2.975 |
| 1.175 | 1.190 | 1.236 | 1.492 | 2.498 | 2.498 | 2.859 | 2.968 |
| 1.156 | 1.146 | 1.330 | 1.564 | 2.475 | 2.477 | 2.974 | 2.979 |
| 1.060 | 1.068 | 1.176 | 1.275 | 2.565 | 2.568 | 2.789 | 2.932 |
| 1.092 | 1.094 | 1.360 | 1.325 | 2.516 | 2.518 | 2.828 | 2.950 |
| 1.081 | 1.088 | 1.296 | 1.232 | 2.506 | 2.504 | 2.844 | 2.958 |
| 1.141 | 1.143 | 1.274 | 1.428 | 2.503 | 2.503 | 2.808 | 2.922 |
| 1.059 | 1.061 | 1.209 | 1.156 | 2.502 | 2.505 | 2.823 | 2.931 |
| 1.106 | 1.109 | 1.253 | 1.350 | 2.518 | 2.521 | 2.792 | 2.990 |
| 1.070 | 1.072 | 1.312 | 1.276 | 2.432 | 2.433 | 2.782 | 2.951 |

It could be seen that the results of generating data and response were in accordance with the desired conditions.

### 3.2 Results of PCA-TOPSIS method

Table 5 shows percentage of the variance of two principal component for four correlations among responses.

TABLE 5
Percentage of the variance of two principal component for four correlations among responses.

| No correlations | | Weak correlations | | Medium correlations | | High correlations | |
|---|---|---|---|---|---|---|---|
| 1st | 2nd | 1st | 2nd | 1st | 2nd | 1st | 2nd |
| 40.39 | 33.08 | 38.19 | 32.90 | 41.68 | 33.80 | 65.14 | 34.28 |
| 34.52 | 33.12 | 40.37 | 32.95 | 37.11 | 33.99 | 61.63 | 35.16 |
| 41.44 | 33.54 | 35.82 | 35.07 | 37.34 | 34.94 | 61.73 | 33.31 |
| 37.83 | 31.87 | 35.99 | 32.47 | 36.49 | 33.60 | 64.35 | 26.46 |
| 35.40 | 34.41 | 41.28 | 32.81 | 39.18 | 32.47 | 72.96 | 20.46 |
| 38.58 | 34.86 | 36.56 | 35.00 | 39.36 | 34.31 | 66.94 | 32.61 |
| 40.69 | 32.03 | 38.30 | 34.02 | 36.58 | 34.25 | 65.42 | 29.90 |
| 42.00 | 32.23 | 37.78 | 33.59 | 41.65 | 31.77 | 67.92 | 24.74 |
| 42.41 | 34.39 | 34.69 | 33.54 | 39.66 | 33.78 | 65.57 | 34.19 |
| 37.03 | 34.85 | 40.51 | 30.78 | 40.09 | 35.06 | 66.66 | 31.40 |

It could be seen that for no correlations among responses, weak correlations among responses and medium correlations among responses two principal components were unable yet to explained all of the variances so that the entire principal components were still remains three. As for high correlations among responses two principal components were able to explained all of the variances so that the principal components were remains two which meant the third variable or third response not included in the analysis using TOPSIS.

Table 6 shows the most maximum OPI for four correlations among responses.

TABLE 6
The most maximum OPI for four correlations among responses

| | OPI | Experimental runs |
|---|---|---|
| No correlations | 0.851 | 10th |
| Weak correlations | 0.961 | 10th |
| Medium correlations | 1.000 | 10th |
| High correlations | 1.000 | 10th |

Based on these results, it could be seen that the method of PCA-TOPSIS was consistent in generating the optimal combination of factor/level in any correlation level, either uncorrelation to high correlation. This meant the level of correlation did not affect the response and still produced same optimal combination factor/level, it was contradicted to the research of Tong et al [4] which stated that correlation could affect response. Figure 2 shows that the value of the most maximum OPI is on the 10th experiment.

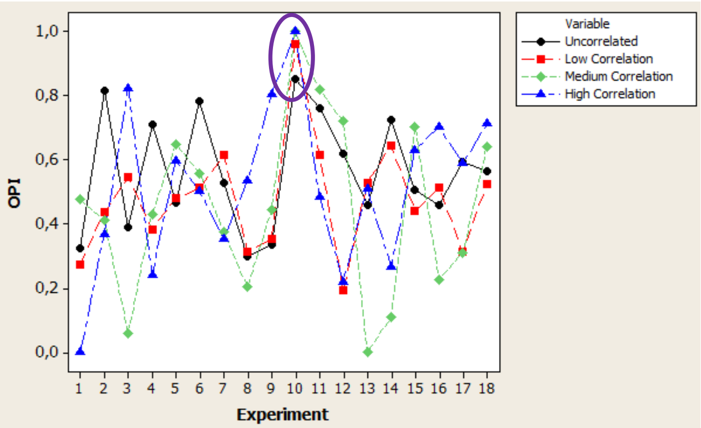


Fig 2. *Line plot* of OPI for all experimental runs on four correlations among responses

### 3.3 Results of VIKOR Method

Table 7 shows the most minimum VIKOR index for four correlations among responses.

TABLE 7
The most minimum VIKOR index for four correlations among responses

| | VIKOR index | Experimental runs |
|---|---|---|
| No correlations | 0.080 | 10th |
| Weak correlations | 0.100 | 10th |
| Medium correlations | 0.030 | 10th |
| High correlations | 0.241 | 10th |

Based on these results, it could be seen that the method of VIKOR was consistent in generating the optimal combination of factor/level in any correlation level, either uncorrelation to high correlation. This meant the level of correlation did not affect the response and still produced same optimal combination factor/level. Figure 3 shows that the value of the most minimum VIKOR index is on the 10th experiment.

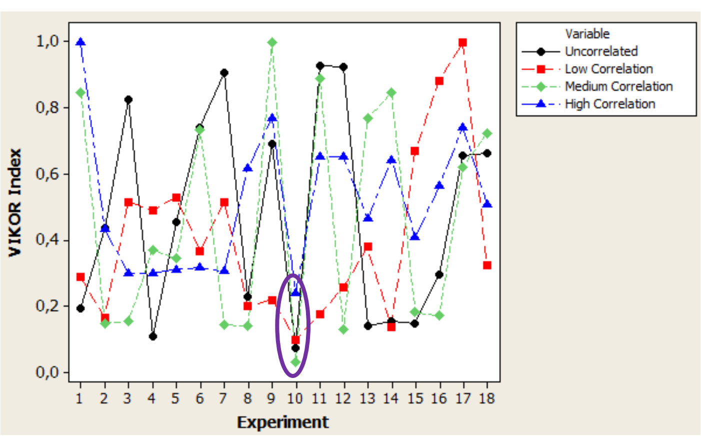


Fig 3. *Line plot of* VIKOR index for all experimental runs on four correlations among responses

Table 8 displays combinations factor/level for each experimental runs.

TABLE 8
Combinations factor/level for each experimental runs

| Ex. no. | Control factor | | | | |
|---|---|---|---|---|---|
| | A | B | C | D | E |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 | 3 |
| 4 | 2 | 1 | 1 | 2 | 2 |
| 5 | 2 | 2 | 2 | 3 | 3 |
| 6 | 2 | 3 | 3 | 1 | 1 |
| 7 | 3 | 1 | 2 | 1 | 3 |
| 8 | 3 | 2 | 3 | 2 | 1 |
| 9 | 3 | 3 | 1 | 3 | 2 |
| 10 | 1 | 1 | 3 | 3 | 2 |
| 11 | 1 | 2 | 1 | 1 | 3 |
| 12 | 1 | 3 | 2 | 2 | 1 |
| 13 | 2 | 1 | 2 | 3 | 1 |
| 14 | 2 | 2 | 3 | 1 | 2 |
| 15 | 2 | 3 | 1 | 2 | 3 |
| 16 | 3 | 1 | 3 | 2 | 3 |
| 17 | 3 | 2 | 1 | 3 | 1 |
| 18 | 3 | 3 | 2 | 1 | 2 |



## 4 Conclusion

Optimization through PCA-TOPSIS and VIKOR in variation conditions correlations among responses shows the same patterns. The optimal solution was the same for all conditions. Hence, correlations among responses do not affect PCA-TOPSIS and VIKOR calculation of responses.

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